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A preliminary study of atmospheric waves using radiosonde data.

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G. A. Corby.

(Meteorological Office)

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

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Introduction

Most of the observational evidence of mountain airflow effects in general, and of lee waves in particular, has come from the pilots of gliders and aircraft. Although such observations have contributed greatly to our understanding of the problem of mountain airflow, their application is nevertheless limited. Furthermore, field investigation of the problem using gliders or aircraft presents formidable difficulties, apart from being extremely expensive. It is natural to enquire, therefore, to what extent the data from routine radiosonde ascents can be used to study atmospheric waves. This paper is concerned with a trial determination and analysis of the vertical air currents revealed by six months radiosonde data from four British upper air stations.

1. The data and their treatment

The data used were from the stations Leuchars, Stornoway, Aldergrove and Liverpool for the six months period November 1953 to April 1954. Both the 0200 and 1400 G.M.T. soundings were used, making a total of about 1440 soundings in all.

The so-called Vaisala diagram which is plotted at British radiosonde stations includes a plot of pressure against time during the ascent. If the rate of ascent of the balloon varies substantially and systematically with height, it is easy to recognise this visually from the pressure-time plot, and an examination of these curves was found to be adequate for the purpose of selecting the cases worth studying in greater detail. In this way, 61 occasions were selected when the rate of ascent of the balloon varied by approximately $\pm$ 300 ft./min. or more; consideration of the errors of estimating vertical currents from radiosonde data indicated that it would not be profitable to study evidence of smaller vertical currents.

Readings of the audio-frequency of the note controlled by the pressure element of the sonde are plotted against time by the operator at intervals of about 1/3 min., the exact interval depending on the rate of revolution of the sonde's "windmill". From these curves the mean rate of change of pressure-cycles, dc/dt, was evaluated over each minute of each sounding, up to 30 min., using a protractor and a conversion table. In similar fashion, the corresponding rates of change of pressure with pressure-cycles, dp/dc, were evaluated from the calibration curves. Multiplication of these two quantities gave the mean rate at which the balloon crossed the isobaric surfaces, dp/dt, over each minute. Values of dp/dt were converted to the vertical velocities, dh/dt, by multiplying by the values of dh/dp appropriate to the mean pressure and temperature over each minute.

The above procedure for the evaluation of the rates of ascent is one of several alternative methods. It was adopted as the best compromise between convenience and accuracy. As would be expected, as the radiosonde is not

designed to measure vertical air currents, the currents which can be detected are limited by errors of technique and measurement. An assessment of the errors is made in the Appendix, where it is concluded that up to the 30th minute of a sounding, vertical air currents exceeding about 120 ft./min. can be detected. (At higher levels the errors of the procedure would increase rapidly).

An attempt was made to seek independent confirmation of the results from the radar data, which comprise readings of range, elevation and azimuth every minute. The Appendix also contains an estimate of the errors in vertical air currents deduced from these data. The errors depend greatly on range and to a lesser extent on elevation, and although accurate vertical velocities can be determined in the first few thousand feet, the errors rapidly increase upwards (e.g., about 600 ft./min., at a range of 30,000 yd.).

Graphs of the rate of ascent against height of balloon were plotted and a mean rate of ascent was inserted by eye in each case. The graphs were then re-plotted in terms of departures from the mean rate of ascent, i.e., vertical air currents. Most of the cases showed evidence, often striking, of vertical currents which were periodic along the path of the balloon and in some cases of substantial magnitude. These were strongly suggestive of gravity waves, presumably of orographic origin, but it must be realised that other factors could give rise to such variations, for example: convective cloud, intermittent icing, or precipitation. Cases when factors of this sort might conceivably have caused the observed variations in the rate of ascent were rejected on the basis of a study of the synoptic charts. There remained after this sifting process 48 cases in which the deduced vertical air motions could be regarded with considerable confidence as being manifestations of gravity waves.

2. Examples of wave motions, and general comments

In Figs.1 to 12 we give some examples of the graphs of vertical velocity of the air plotted against height of balloon. Each graph comprises two curves, the thick curve being that deduced from the pressure observations of the radiosonde and the thin curve that from the radar data. The areas between the lines AB, CD for the pressure curves and between OX, OY for the radar curves, define the zones in which individual values of vertical velocity are not significant, on the basis of the assessment of errors given in the Appendix. Within these zones the curves are in dashed form. As already indicated, the radar data are more sensitive to vertical currents in about the first five thousand feet, but above this level the pressure data are much more reliable for the purpose in hand. Where a number of successive values are consistently of the same sign they may be significant even if within the "zone of no significance" because the latter zone applies strictly to individual values. In Figs. 4 and 5, the asterisk (*) after the time indicates that the radar data were derived from a separate sounding launched an hour or so later than that from which the pressure data were obtained.

The examples, which are typical of the 48 occasions found, show clear evidence of vertical currents which, as measured by the balloon during its oblique ascent, were periodic in the vertical. Much of the curves obtained from the pressure data is outside the zone of no significance and even where the curves lie within the zone there is in general a good measure of consistency with the remainder of the pattern. Furthermore, the corresponding curves from radar data confirm to a convincing degree the pattern of vertical motion in the lower troposphere, at least as regards the general form if not always the magnitude. It may also be mentioned that on about half the 48 occasions there was confirmation in the form of aircraft reports of waves viz., the reports collected at Northolt Airport for the same period and used by Pilsbury (1955) in an analysis of aircraft reports of waves. There is, therefore, no reason to doubt the reality of the waves deduced here from radiosonde data.

In the majority of cases, the amplitude of the vertical velocity perturbation attains a pronounced maximum in the lower troposphere (usually from 5,000 - 10,000 ft. above the ground) and decreases sharply above. Sometimes, however, the amplitude is maintained to considerable heights, as for example in

/Figs. 7

Figs. 7 and 11. In no case was there convincing evidence of the amplitude increasing with height in the upper troposphere.

In some cases waves were present on two or more successive ascents, e.g., in Figs. 4 and 5, and also at more than one upper air station, showing that at least sometimes waves have a fair degree of persistence in space and time. It is interesting to note, too, the agreement on Fig. 5 between the radar and pressure data which were not in this case simultaneous. On the other hand, Fig. 4, for which the two sets of data were also not simultaneous, shows differences between the heights of the crests and troughs; such differences could easily be explained in terms of small changes in wavelength affecting the location of crests and troughs relative to the point of release of the balloon.

The case which gave rise to the largest amplitude waves (with vertical velocities of about 1,000 ft./min.) is included in these examples, viz., Fig. 8, referring to 15th April 1954. For two or three days about this time waves were widespread and were the subject of many aircraft reports, one of which refers to vertical currents of 1,900 ft./min. in the region of Edinburgh (see Pilsbury, 1955). In addition, it may be noted that the violent turbulence encountered by a Canberra at 40,000 ft. near Edinburgh and described by D.C.E. Jones (1955) occurred on the evening of 14th April 1954. Strong waves were evident from the Leuchars sounding nearest in time (1400 G.M.T., 14th April 1954) although not as large in amplitude as those on Fig. 8.

3. Some statistical results

(a) Distribution amongst stations

The 48 cases were divided amongst the stations as follows:-

Station	No. of cases
Leuchars	27
Liverpool	14
Stornoway	4
Aldergrove	3

We should expect the frequency of detectable waves to be closely dependent on the character of the topography upwind of the stations in the direction of the prevailing wind and this no doubt accounts for the wide variation in the frequency of waves at the four stations.

(b) Distribution of maximum vertical currents

The frequency distribution of the maximum vertical currents, based on the pressure frequency data, is illustrated by the histogram at Fig.13(a). As expected, the distribution is J-shaped indicating increasing rarity the larger the vertical current.

The distribution of the heights at which the balloon encountered the maximum vertical current is given in Fig.13(b). The main feature is the pronounced maximum in the layer 5,000 to 10,000 ft. Generally speaking, the theory of mountain airflow leads us to expect the amplitude of lee waves to increase upwards to some level in the lower troposphere and to decrease at higher levels. This picture is supported by the present data. It is not thought that the minor secondary maximum from 20,000 to 25,000 ft. is significant.

(c) Seasonal and synoptic variations

The occasions of waves were distributed amongst the six months as follows:-

Month	No. of cases
November 1953	10
December 1953	2
January 1954	14
February 1954	4
March 1954	10
April 1954	8

/We

We know that the winter produces more situations favourable for waves than other times of the year because of the greater frequency in winter of strong winds and high static stability in the lower troposphere. However, the only significance to be attached in the above table to the low frequency of waves in December 1953 and February 1954 probably arises from the variability of the British climate. These particular two months were on the whole much quieter and less windy than usual over Britain.

The theoretical condition for lee waves deduced by Scorer (1949) is not one which can be applied with precision in practice and it is well known that the condition can be satisfied in a wide variety of synoptic situations. However, in a high proportion of the present cases, viz. 30 out of 48, the radiosonde station was near a front - sometimes ahead of a warm front or near an occlusion, but more commonly to the rear of a cold front. Such a location generally provides the stability in the lower troposphere and the increasing winds with height, both of which tend to favour waves. Furthermore, it was clear from the time sequence of the occasions of waves at the different upper air stations that the region relative to the front where waves occurred tended to be advected with the front and to be maintained with it during the passage of the front across the British Isles. The remaining cases mostly occurred in warm sectors or anticyclonic westerlies both of which are characterised by low level stability and increasing winds with height.

(d) Diurnal variations

It has been suggested on theoretical grounds (see Scorer, 1953) that lee waves should be more common by night than by day, the reason being the greater tendency at night for high static stability and laminar flow in the low levels. In the present examples, however, the reverse occurred, 34 cases being from 1400 G.M.T. soundings and only 14 from the 0200 G.M.T. ascents. This apparent discrepancy probably arises because the theoretical conclusion is concerned only with the occurrence of waves irrespective of their amplitude. In a discussion of the amplitude of lee waves, Corby and Wallington (1955) indicated that a falling off of surface wind such as occurs at night should be accompanied by a reduction in the amplitude of any lee waves, although it might also result in waves when these were not previously operating. Thus waves of unspecified amplitude may well be more common by night than by day, but in the present study waves of small amplitude were necessarily excluded.

(e) Wind speed and direction

In Figs.14(a) and (b) are given in the form of polar diagrams the 900 mb. wind speeds and directions at Leuchars and Liverpool for the wave cases. The mean 900 mb. wind speed was 32.1 kt. at Leuchars and 28.4 kt. at Liverpool; these appear to be a little above normal for the winter months, although not greatly so.

As regards wind direction, all the Leuchars cases except two occurred with winds between N and S through W, i.e., directions with high ground upwind. In one case which might appear suspicious, viz., that with a NE wind, the wind below 900 mb. was well backed and was sharply anticyclonic with Westerlies in extreme N. Scotland; the low-level flow had, thus, a predominantly land track. The largest amplitude waves were concentrated in the NW'ly quadrant where the highest mountains, upwind from Leuchars, exist.

The wind directions at Liverpool, Fig.14(b), were much more varied, as might be expected in view of the local topography. We will refrain from speculating as to whether waves with NW'ly winds owed their origin to the effect of the Isle of Man or to the hills of Northern Ireland, but will remark that lenticular clouds have been observed from the Lancashire coast stretching across the Irish Sea.

Of the 48 cases as a whole, the change in wind direction from 900 to

/500 mb.

500 mb. was less than 20 deg. in 36 cases and in only 2 did it exceed 30 deg. This confirms the well-established conclusion that relative constancy of wind direction with height is required for well developed lee waves.

The four cases at Stornoway all occurred with strong SSW'ly winds; the airflow would thus have passed over the high ground in the SSW of the island of Lewis itself. The Aldergrove examples were all with NW'ly winds.

(f) The wavelength

In many of the graphs of vertical motion against height of balloon, of which Figs. 1-12 are examples, a dominant wavelength stands out unmistakably. In fact, it was found possible in 26 cases to obtain a good estimate of the dominant wavelength, as the average of about 5 or 6 assessments over different portions of the curve, assuming vertical wave fronts. The average wavelength for the 26 cases was 6-7 mi. (S.D. 3 mi.), the extreme values being 2.6 mi. and 14.4 mi. These values are consistent with observational evidence and with the order expected from theoretical considerations.

Perturbation theory enables us to set down equations which formally determine the wavelength, but a general analytical solution is not possible owing to the complex way in which variations of static stability and wind in the vertical are involved. However, it seems clear that generally speaking the wavelength should increase with the mean wind speed through the troposphere. This conclusion is strikingly borne out by Fig. 15 which is a plot of the wavelengths in the 26 cases against the corresponding wind speed meaned over the layers contributing to the wavelength assessments. The correlation coefficient is 0.91 which is sufficiently high to justify estimates of wavelength from mean tropospheric wind speed alone.

In Fig. 16 is plotted the maximum vertical component of wind in the Leuchars examples against the wavelength, when an estimate of wavelength was possible. Clearly there is no linear relation; indeed there are signs of a peak maximum vertical velocity for a wavelength of about 8 mi. with smaller vertical velocities for both shorter and longer wavelengths, and a rough curve has been drawn by eye expressing this trend. This appears to offer some support for the conclusion reached by Corby and Wallington (1955) viz., that a kind of resonance effect would be expected to occur when the natural wavelength of the airstream matched the dominant wavelength component of the topography. Admittedly the points on Fig. 16 represent a variety of wind directions and therefore the support is by no means conclusive; sufficient data to enable such graphs to be plotted for distinct wind directions should throw more light on this aspect.

4. Apparent distortion of temperature structure

It was pointed out by Scorer (1953) that if a radiosonde balloon passes through waves, the temperature structure measured by the sonde will differ from that of the undisturbed stream and, indeed, that spurious inversions might appear on this account. If static stability is expressed, in the usual way, as $\theta^{-1}\,\partial\theta/\partial z$ where θ is potential temperature, and we neglect perturbations of horizontal velocity, then it may readily be shown that the apparent stability will be $W/(W + w)$ times the true stability, where W is the rate of ascent of the balloon relative to the air and w is the vertical component of wind. Thus in an ascending current stability will appear to be reduced and vice versa. In the extreme case of a descending current which brought the balloon to rest (in the vertical), the sonde would indicate an ideally sharp inversion whatever the undisturbed lapse rate.

The above remarks prompt us to enquire to what extent the temperature structure measured by a radiosonde passing through waves is in fact distorted in such ways.

/If

If we represent the displacement of a streamline from its undisturbed level by

$$\zeta = a \sin \frac{2\pi}{L}(x + \epsilon) \qquad (1)$$

where a = semi-amplitude

L = wavelength

and ϵ is a phase angle

then $$\frac{\partial \zeta}{\partial x} = \frac{w}{U} = \frac{2\pi}{L} a \cos \frac{2\pi}{L}(x+\epsilon) \qquad (2)$$

and $$[w]_{max} = \frac{2\pi U a}{L} \qquad (3)$$

In a number of cases an estimate of the wavelength, L, was made, values of $[w]$ max. were read from curves such as Figs. 1 to 12, and thus from Eq.(3) the semi-amplitude, a, was obtained at several levels. The value of a at other levels was obtained by interpolation and the displacement at any level deduced from Eqs. (1) and (2). This provides a basis for correcting the temperature structure, subject to the important assumption that the centre line of the wave form at any level is the same as the corresponding undisturbed level upstream. Several tephigrams were corrected in this way and three examples are given in Figs. 17 to 19. In these diagrams the temperature structure as measured by the sonde is plotted as a full line and the corrected version is left dashed. Layers of steep lapse rate are not, of course, much affected but layers of marked stability and in particular inversions may be modified considerably. If a layer containing an inversion in the undisturbed stream is subjected to a maximum vertical current in the wave system as the balloon passes through it, the intensity of the inversion will be modified; if the balloon passes through such a layer in the crest of a wave there will be little effect on intensity but the temperature level will appear to be lower, whilst if measured in a wave trough the temperature will be higher. There is evidence of all these effects in the examples of Figs. 17-19, but it is emphasised that the precise effect in any given case must be almost fortuitous; release of the balloon a mile or two up- or downstream would change the measured temperature structure.

In no case examined in this way did it appear that any of the substantial inversions, which are commonly found when lee waves occur, were completely spurious. The main effect seems to be to change the intensity or level of strong inversions. The dew point curves are modified in a similar fashion.

Conclusions

Where upper air stations are to the lee of hills or mountains, the routine radiosonde data can provide a valuable means of studying orographic lee waves, and of building up background statistics of such phenomena. In the present preliminary study, support was found for several conclusions which have been deduced from perturbation theory. The results obtained here suggest that profitable work might be done by using a mobile radiosonde station amongst mountains. Furthermore, useful indications of the occurrence of waves could be given from existing radiosonde stations on a current basis; at present little or no observational data on waves reaches aviation forecasters, and radiosondes could be exploited to help fill this gap.

/References

References

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Jones, D.C.E.	1955	"Exceptionally severe clear-air turbulence and other phenomena 14th April, 1954". Met. Mag., 84, p.107.
Pilsbury, R.K.	1955	"Standing-wave reports received at Northolt during the winter of 1953-54". Met. Mag., 84, p.313.
Scorer, R.S.	1953	"Theory of airflow over mountains: II - the flow over a ridge". Quart. J.R. Met. Soc., 79, p.70.

Appendix

Assessment of errors in the evaluation of rates of ascent from radiosonde records

1. Method using pressure data

In assessing the errors of our procedure we are not concerned with the errors in the measurement of pressure from one sonde to another, that is with the absolute accuracy of pressure readings, but with the effect of random errors between successive readings during a single ascent.

Let p = pressure
c = corresponding pressure-cycles
t = time
h = height
T = temperature.

Our procedure is to evaluate dp/dc from the calibration curve and multiply this by the corresponding value of dc/dt from the plot of pressure-frequency against time. The result is dp/dt which is converted to dh/dt using a table giving dh/dp in terms of p and T. We ignore the very small systematic error due to the temperature correction to the calibration of the pressure element.

(i) Error in dp/dc

Denoting the slope of the calibration curve by θ so that $dp/dc = k \tan \theta$ then if θ is measured by protractor to the nearest degree, the maximum error in θ will be $\frac{1}{2}$ deg. or, say, 1/100 radian. Then maximum error in $dp/dc = \frac{1}{100} \times k \sec^2 \theta$ and maximum percentage error in $dp/dc = 2 \operatorname{cosec} 2\theta$ %.

Now, typical values of θ are about 55 deg. at release decreasing upwards to about 35 deg. by the 30th minute. Thus the maximum % error in dp/dc over the first 30 minutes of a sounding varies in the range 2.0 to 2.1% and for our present purpose we shall use 2% throughout.

(ii) Error in dc/dt

Values of the pressure-cycles, c, are plotted to the nearest $\frac{1}{2}$ cycle and this is understood to be well justified by the stability of the pressure element. Thus the maximum error in a single reading is $\frac{1}{4}$ cycle. The error in placing the curve in the vicinity of an individual point is, however, less than this because neighbouring points have an influence. It seems appropriate to take the maximum error in a single value read from the curve as $1/4\sqrt{2}$ cycles. If the slope of the curve of pressure frequency is measured over a one-minute interval, the maximum error in dc/dt will therefore be $1/2\sqrt{2}$ cycles/minute. Hence the maximum percentage error in dc/dt due to errors in the values of c will be $\frac{100}{2\sqrt{2}\, dc/dt}$ %.

There is also an error due to the uncertainty of timing. Readings are plotted to the nearest 1/20 min., so that the maximum error of an individual reading is 1/40 min. Hence on the same basis as above, the maximum % error in dc/dt due to timing errors will be $100/20\sqrt{2}$%.

There remains the error in the measurements by protractor. Assessed as in (i) above this leads to a range of values for the maximum error in dc/dt of 2.4 to 2.5% and we shall use 3% throughout.

(iii) Total error of the combined procedure

We take the square root of the sum of the squares of the component errors to obtain as the total error of the combined procedure:-

$$\left\{ (2)^2 + \left(\frac{100}{2\sqrt{2}\, dc/dt}\right)^2 + \left(\frac{100}{20\sqrt{2}}\right)^2 + (3)^2 \right\}^{1/2} \quad \%$$

Now, dc/dt has typical values as follows:-

From release	to 15 min.	5.0	cycles/min.
From 16	" 20 "	4.7	" "
From 21	" 25 "	4.2	" "
From 26	" 30 "	4.0	" "

Thus, departures from the mean rate of ascent having percentage values less than the following could arise due to errors of measurement and of the method:-

From release	to 15 min.	8.7%
From 16	" 20 "	9.1%
From 21	" 25 "	9.8%
From 26	" 30 "	10.2%

Departures from the mean rate of ascent exceeding these percentage values should have real significance. This, of course, relates to a single value for the rate of ascent evaluated over a 1 minute interval. If n successive departures had the same sign, their mean would be significant if it exceeded the above values divided by $\sqrt{n}$.

2. Method using radar data

Let R = true range and
E = true elevation.

Then true height $h = R \sin E$.

The measured height $(h + \delta h) = (R + \delta R) \sin (E + \delta E)$

where δR, δE are the errors in range and elevation and δh is the corresponding height error.

Thus the height error of a single observation
$= (R + \delta R) \sin (E + \delta E) - R \sin E$

$\simeq \delta R \sin E + R \delta E \cos E$

$\simeq R \delta E \cos E$ since this term will dominate.

Hence the probable error in the difference between successive readings is $\sqrt{2}.R \delta E \cos E$ where δE is now the probable error in the elevation of a single observation.

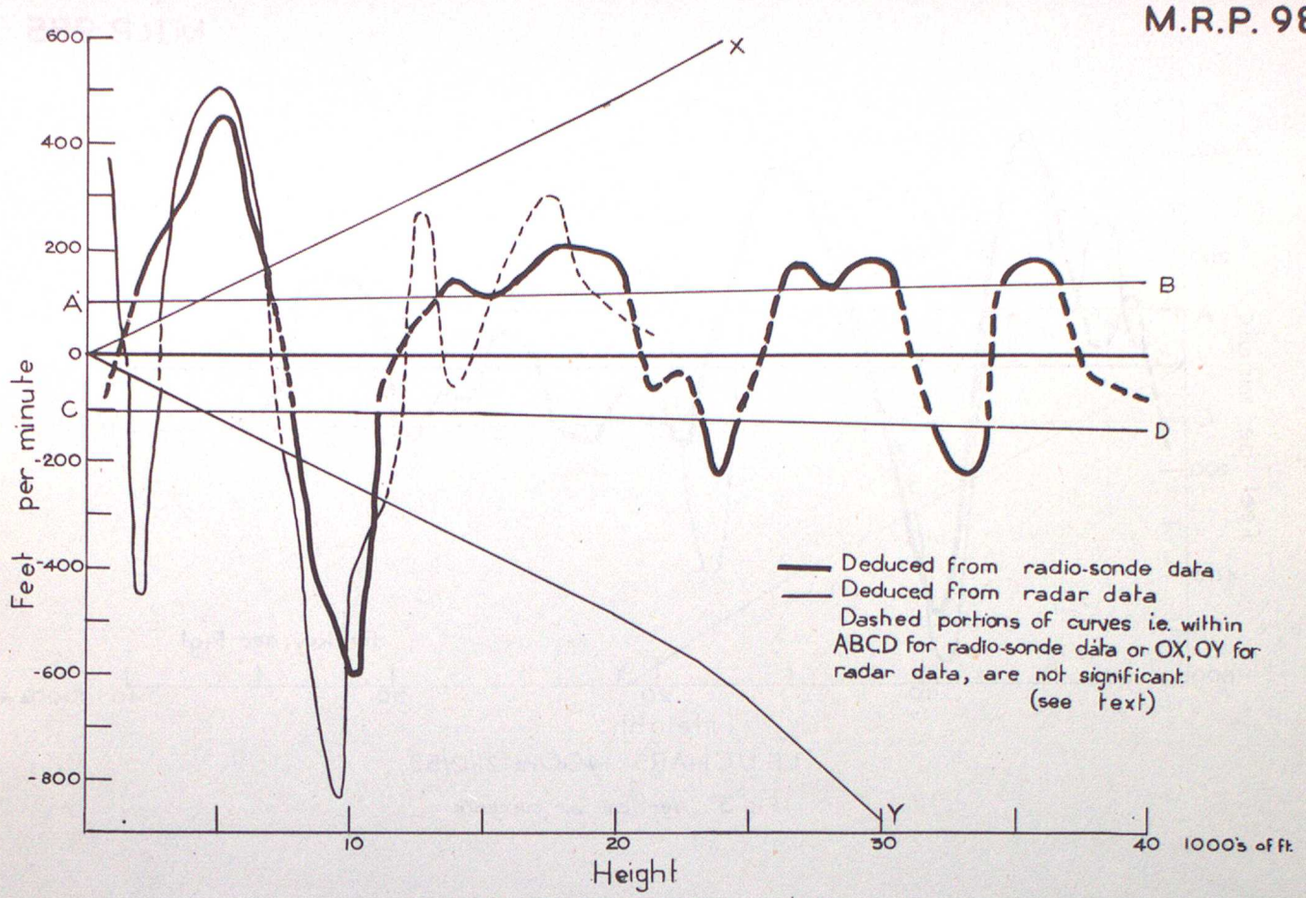
Now $\delta E = 0.1$ deg. $\simeq 1/600$ radians (A.P. Taylor, unpublished).

The radar observations are made at 1 min. intervals and thus rates of ascent calculated from successive radar readings will be significantly different from the mean rate of ascent (at the 5% level) if the difference from the mean exceeds $\frac{\sqrt{2}}{200} R \cos E$.

Typical vertical air currents detectable from radar heights will thus be of the following order:-

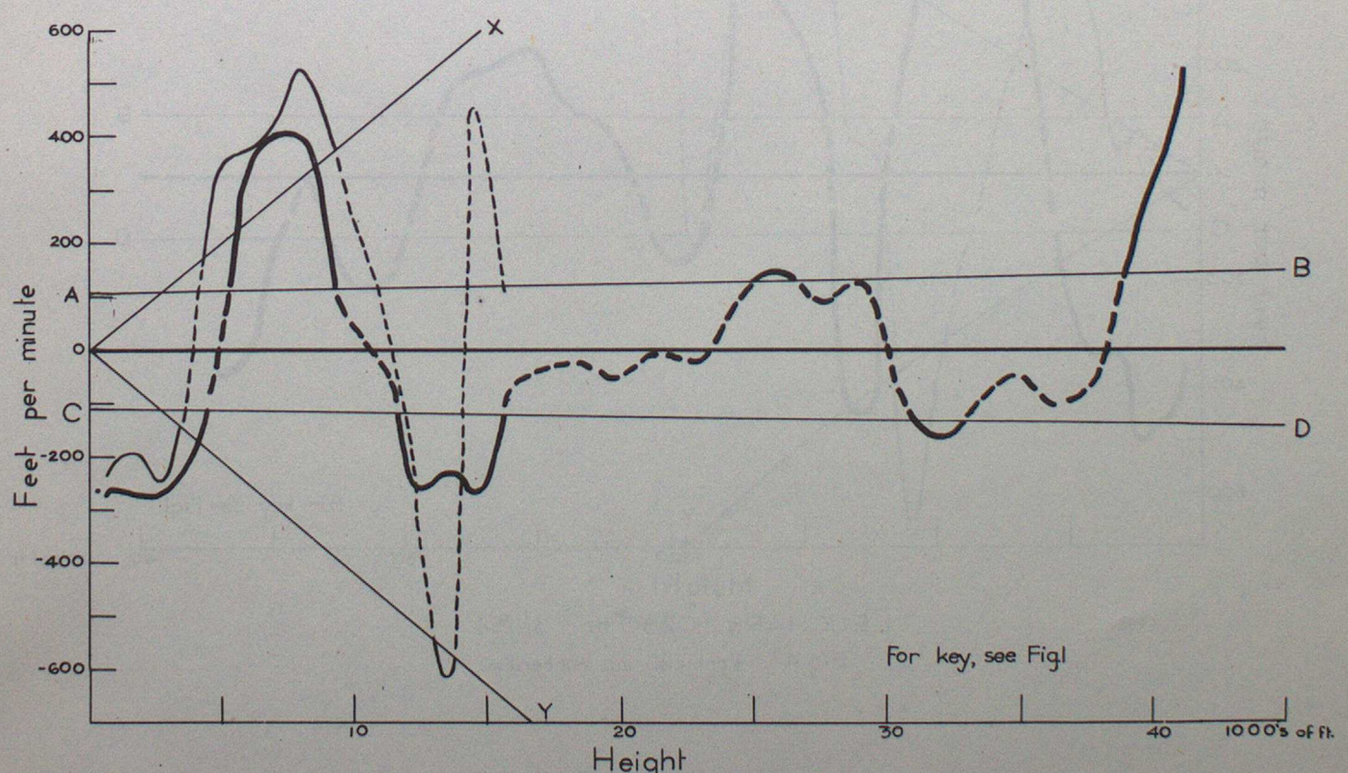
Radar Range	(30,000 yd.	- 600 ft./min.
	(60,000 yd.	- 1200 ft./min.
	(90,000 yd.	- 1800 ft./min.

The variation from sounding to sounding is, however, quite wide and actual values of R and E were used in the paper to assign the limits OX, OY on Figs. 1 to 12.



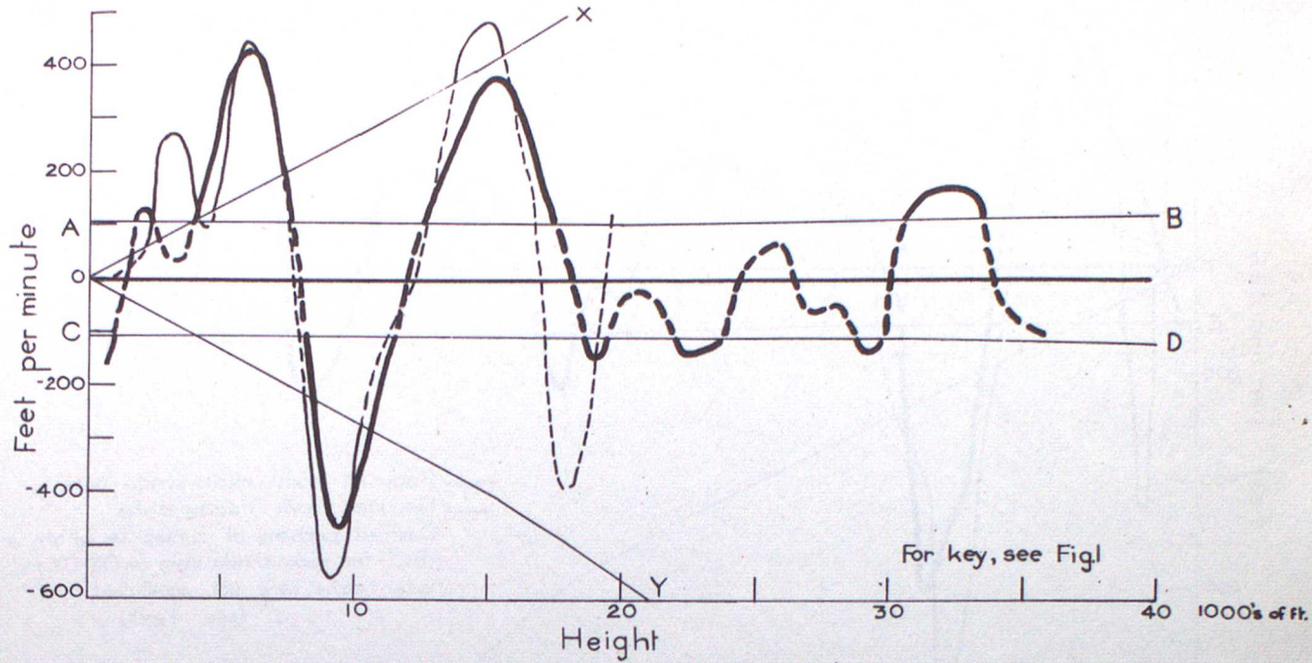
LEUCHARS 1400hr. 4/11/53

Fig.1 Vertical air currents



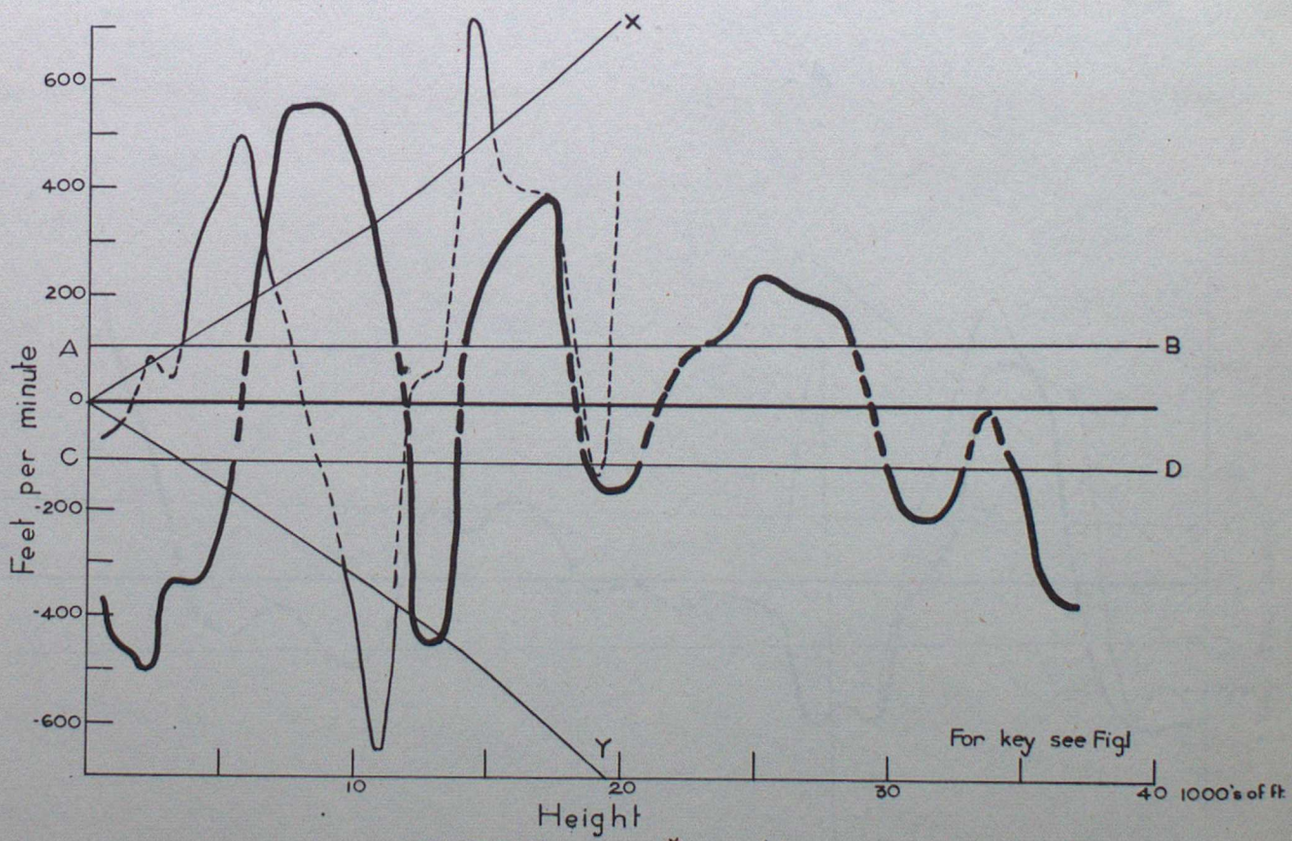
LEUCHARS 0200hr. 30/11/53

Fig.2 Vertical air currents



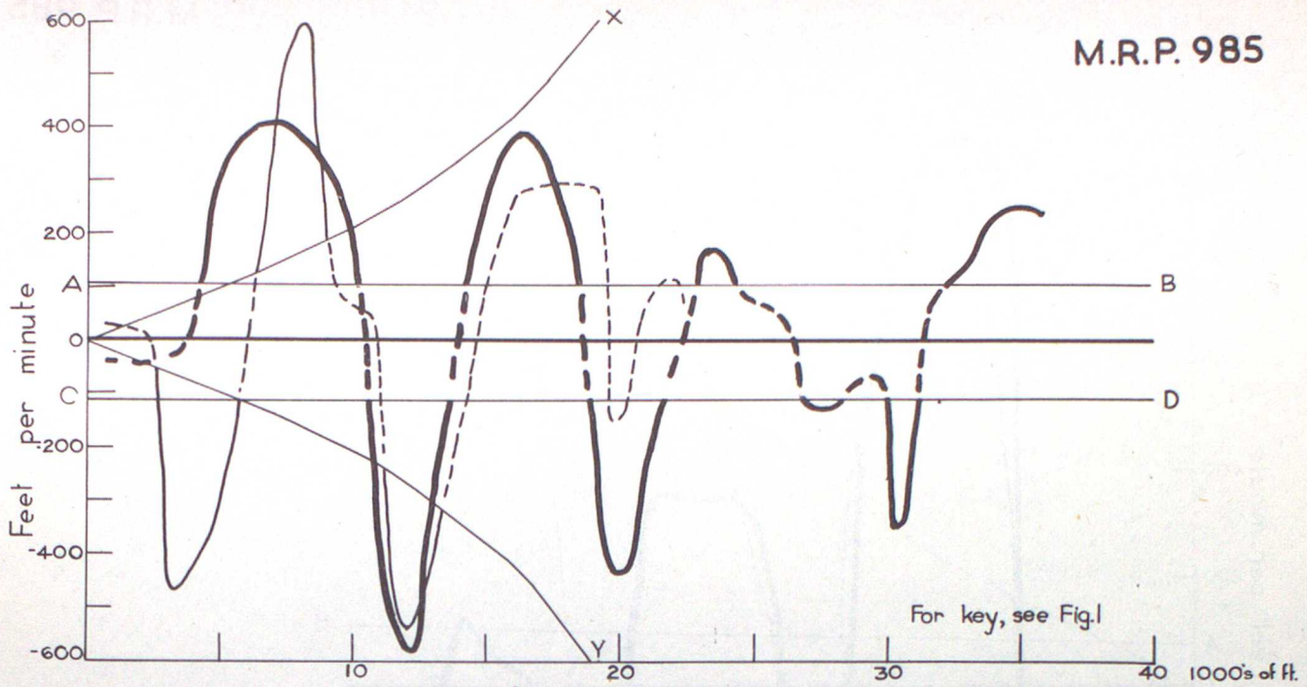
LEUCHARS 1400hr 21/12/53

Fig. 3 Vertical air currents



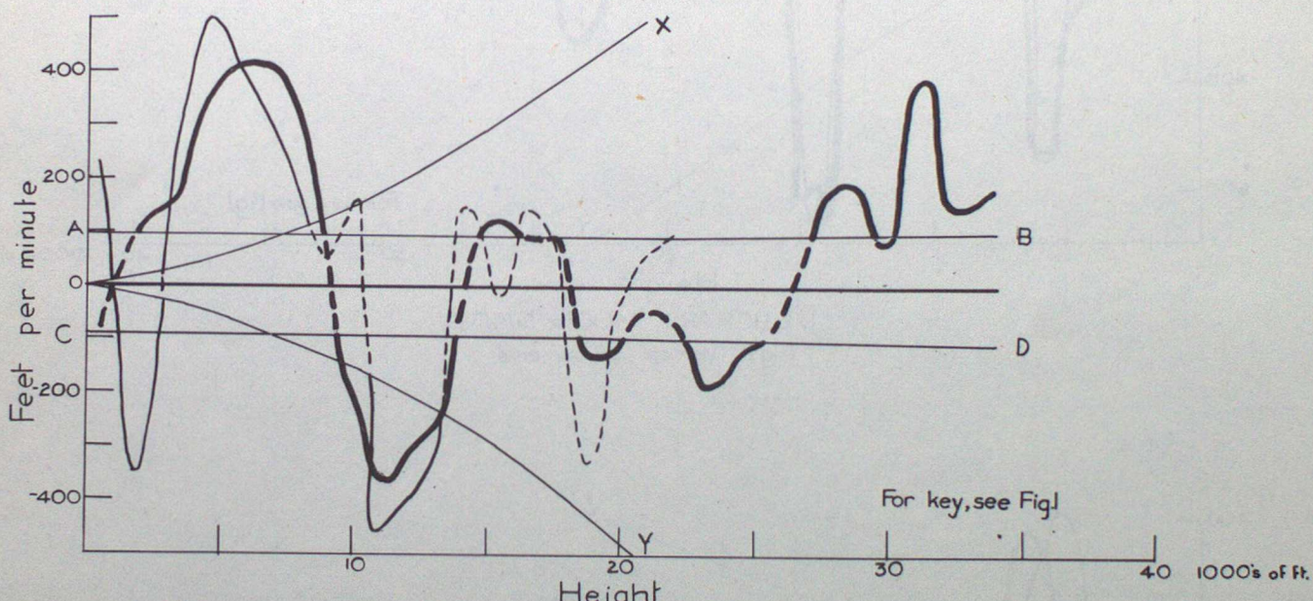
LEUCHARS 0230* hr. 3/1/54

Fig. 4 Vertical air currents



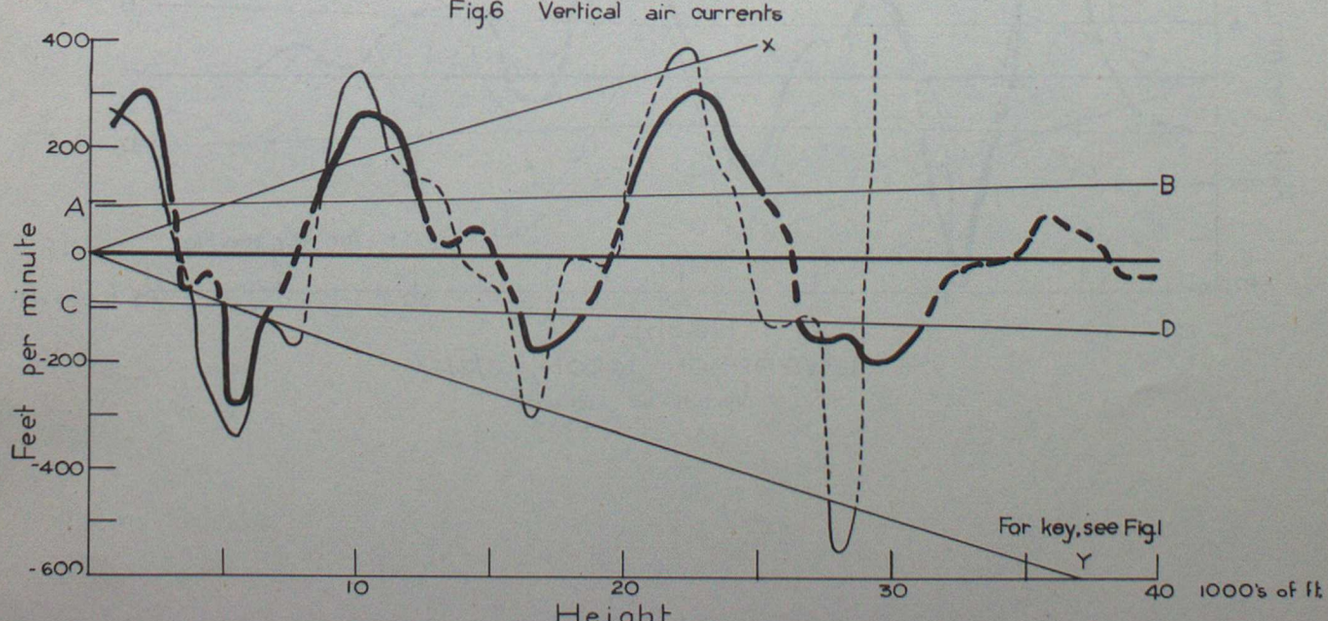
LEUCHARS 1445* hr. 3/1/54

Fig.5 Vertical air currents



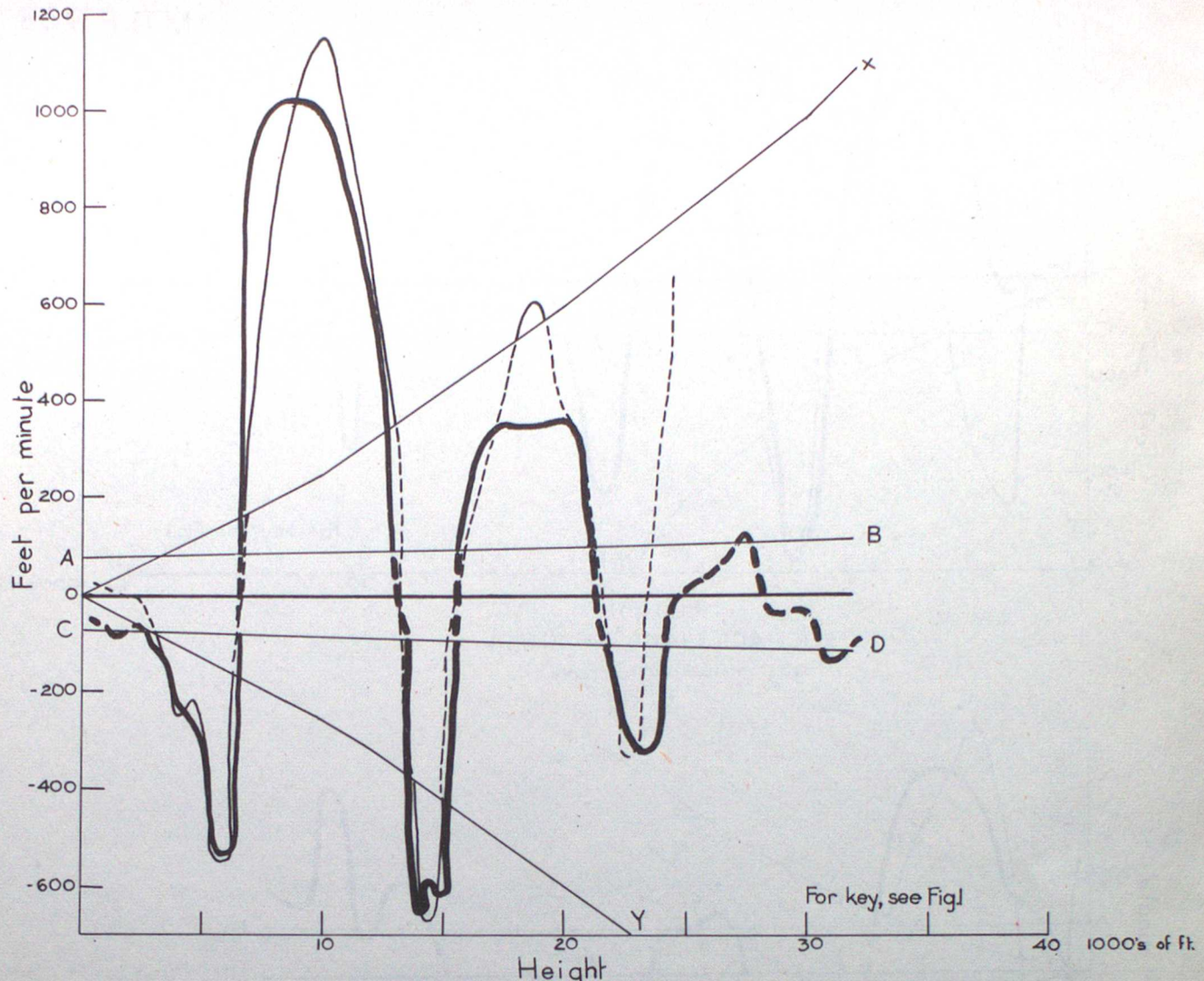
LEUCHARS 1400hr. 5/3/54

Fig.6 Vertical air currents



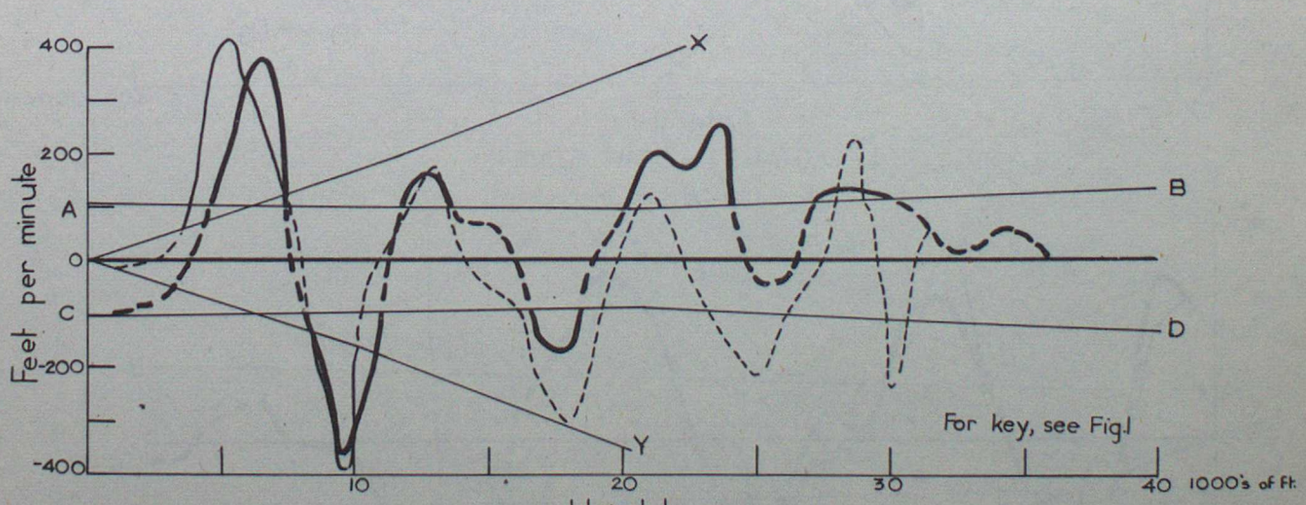
LEUCHARS 1400hr. 11/4/54

Fig.7 Vertical air currents



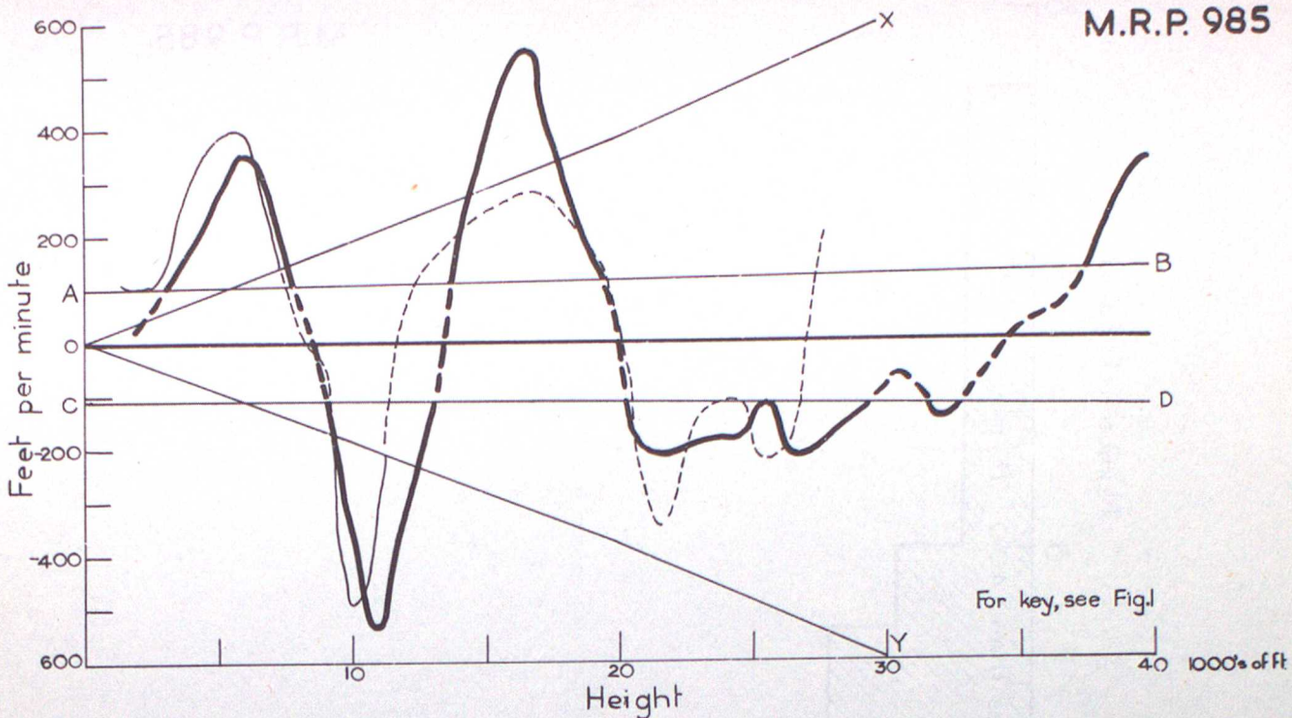
LEUCHARS 1400hr. 15/4/54

Fig.8 Vertical air currents



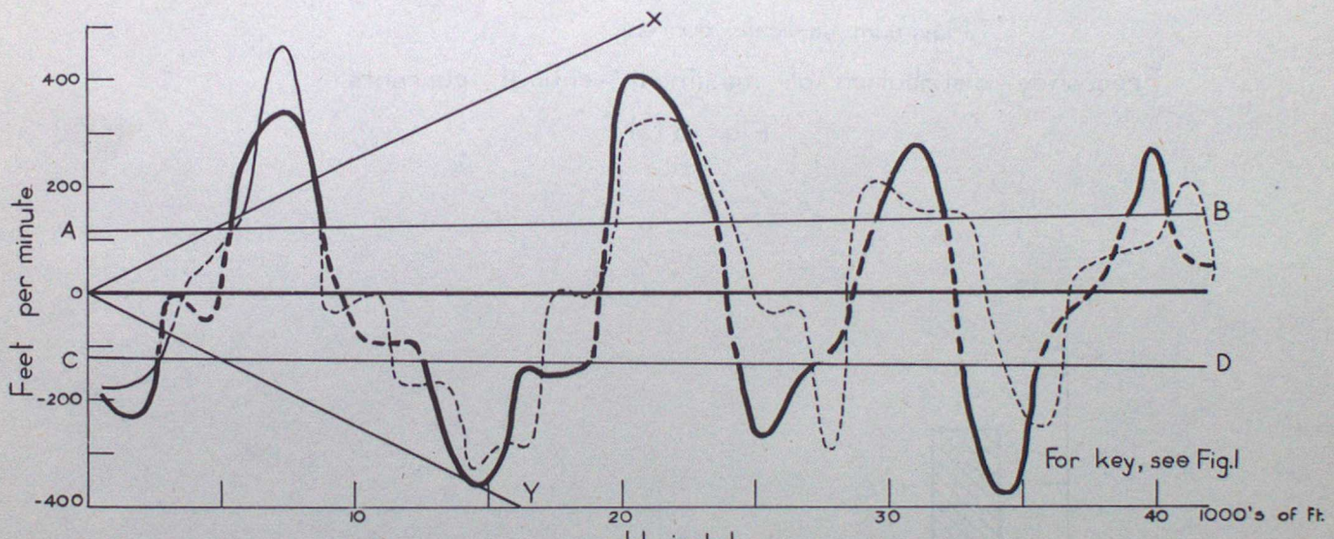
LIVERPOOL 1400hr. 2/2/54

Fig.9 Vertical air currents



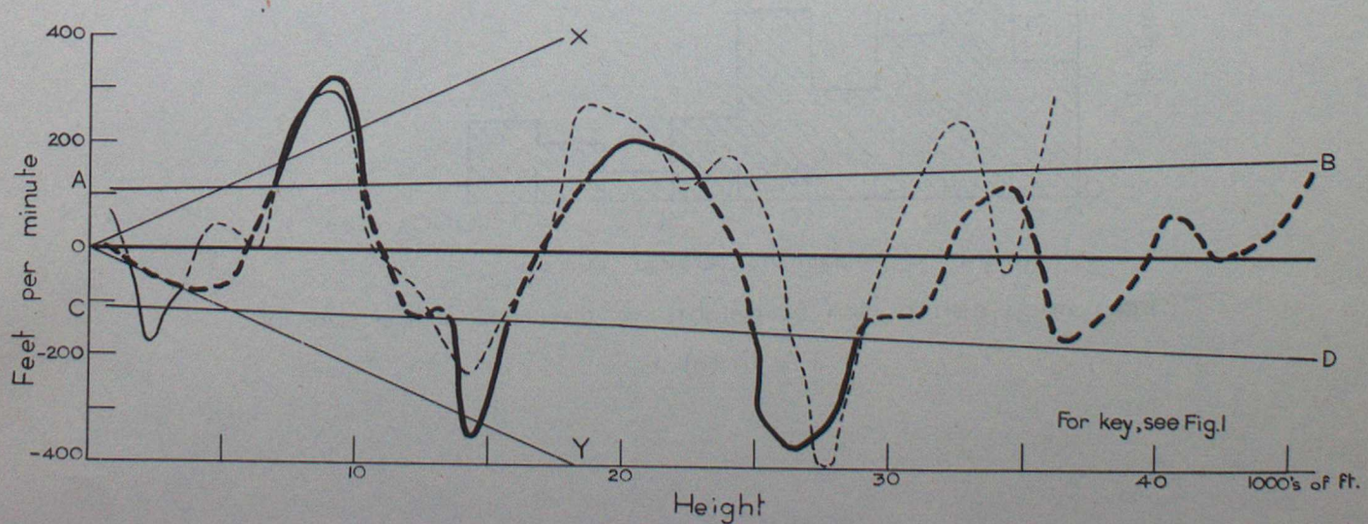
LIVERPOOL 1400 hr. 21/3/54

Fig.10 Vertical air currents



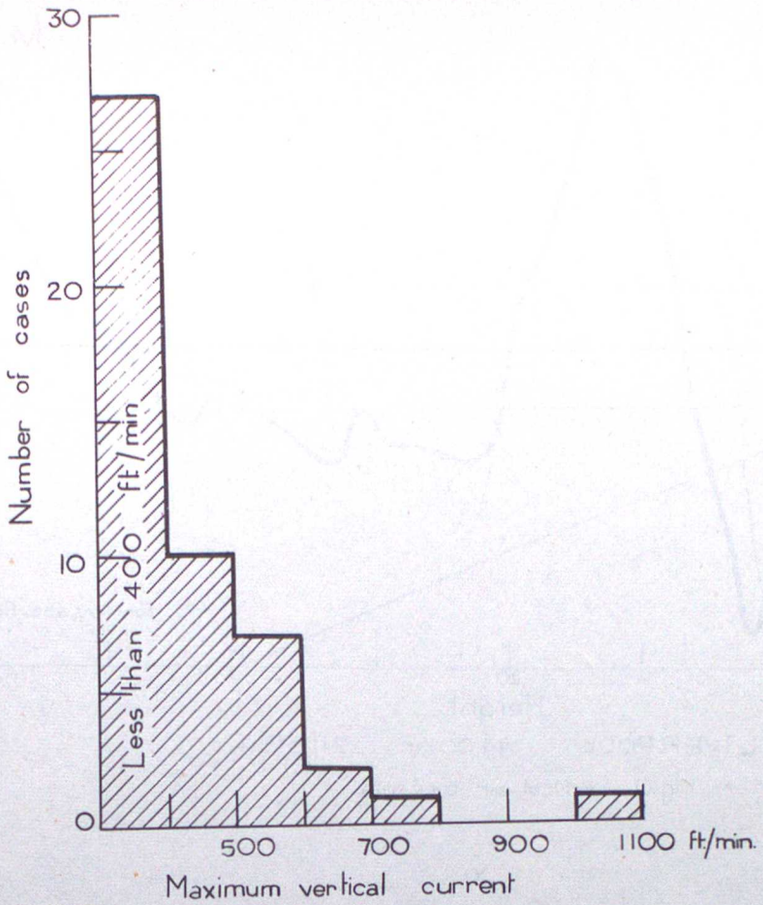
STORNOWAY 0200 hr. 23/11/53

Fig.11 Vertical air currents



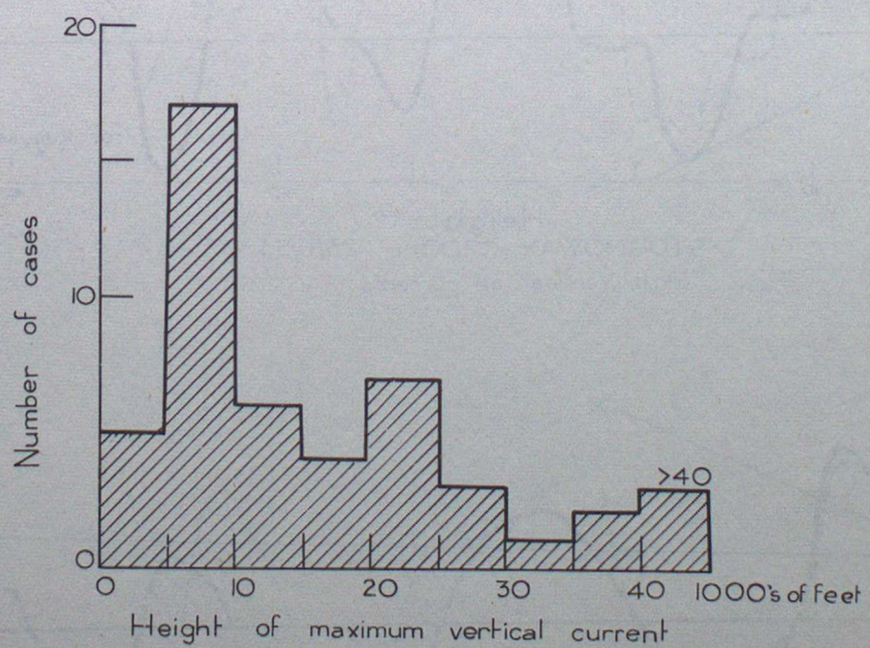
ALDERGROVE 0200 hr. 15/4/54

Fig 12. Vertical air currents



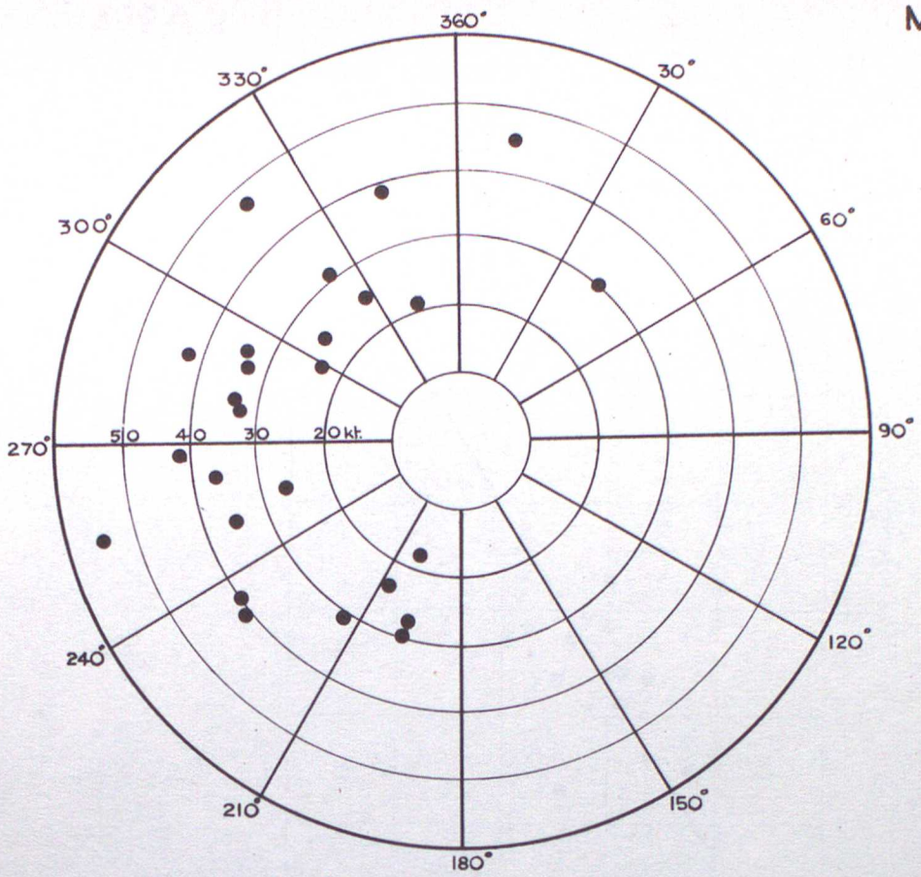
Frequency distribution of maximum vertical currents

Fig. 13 (a)



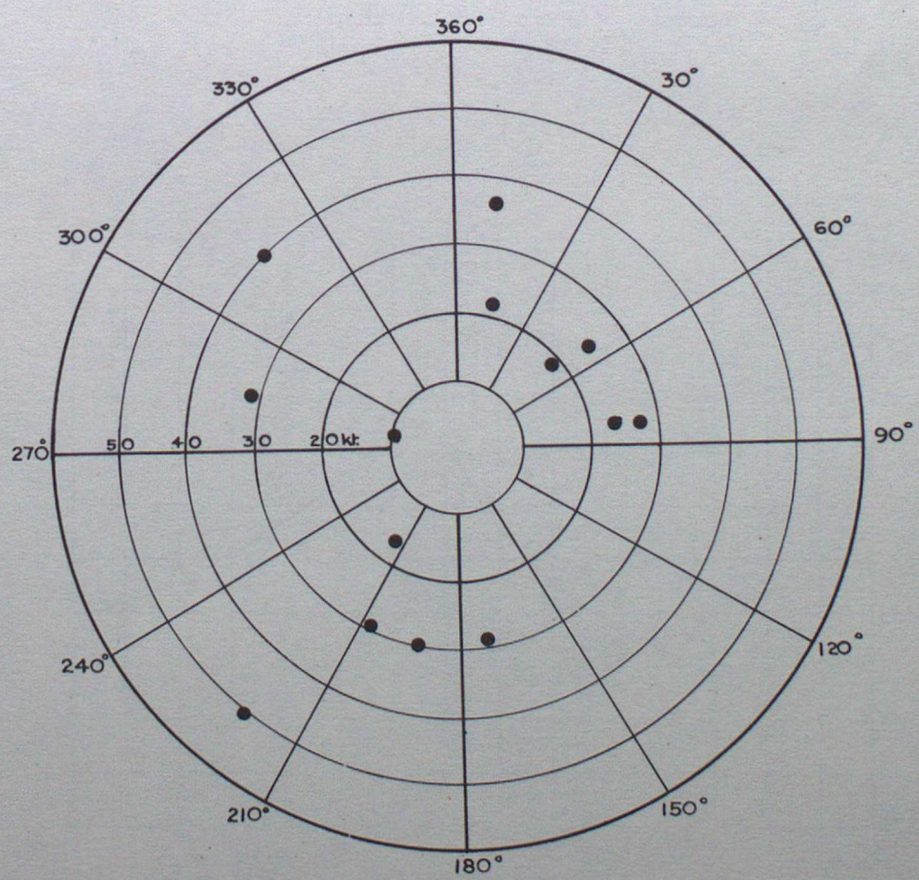
Frequency distribution of heights of maximum vertical current

Fig. 13 (b)



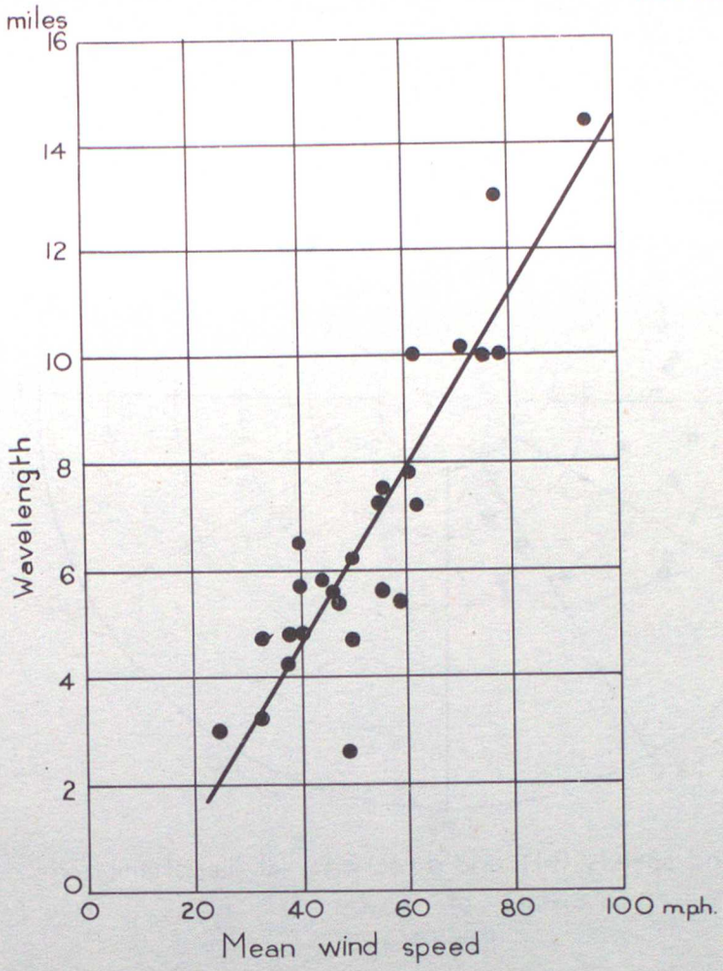
900 mb. wind speeds (kt.) and directions at Leuchars for cases of waves

Fig. 14(a)



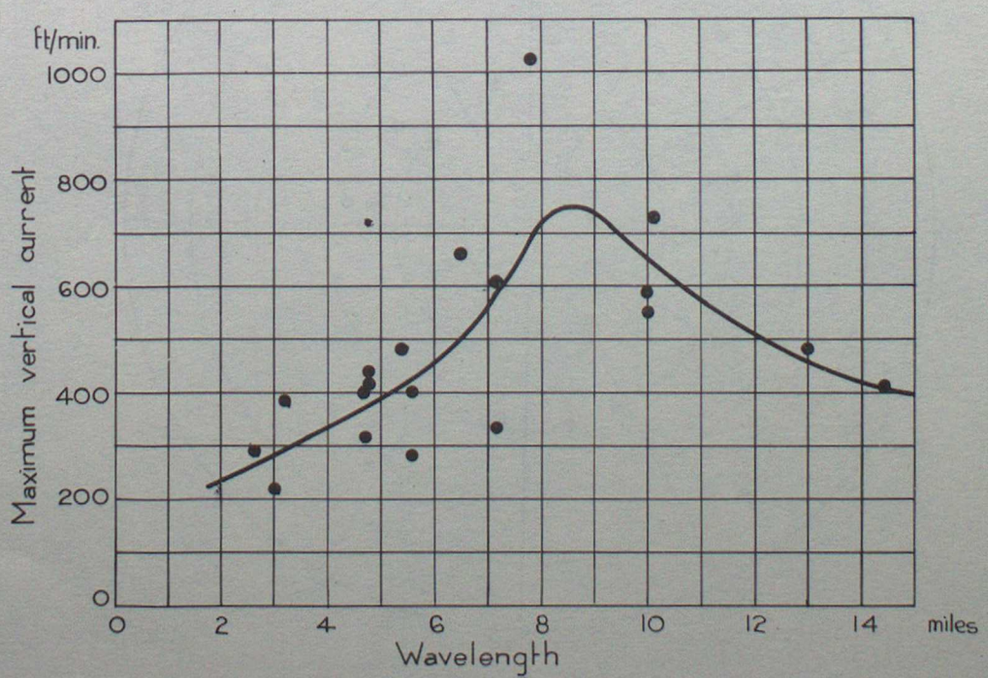
900 mb. wind speeds (kt.) and directions at Liverpool for cases of waves

Fig. 14(b)



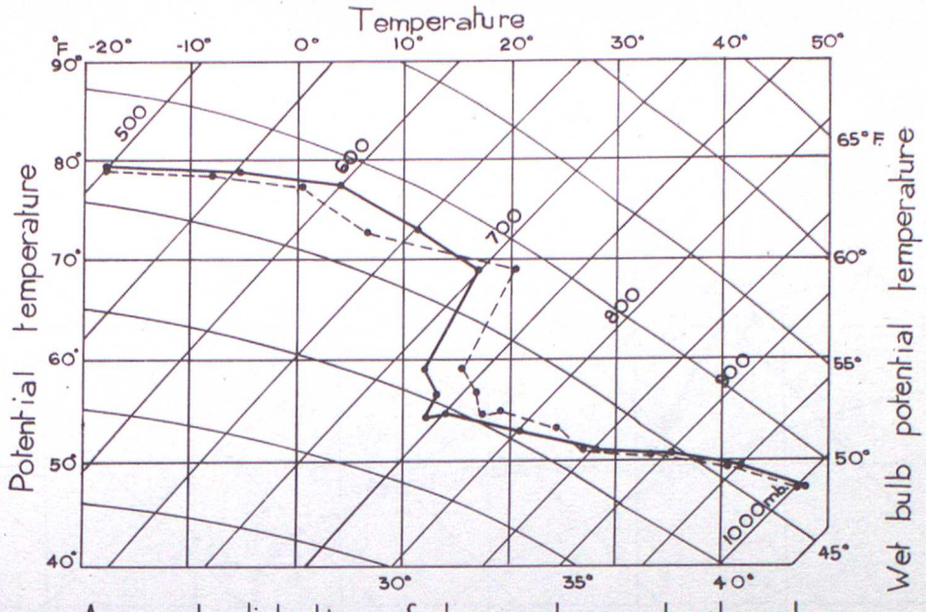
Relation between wavelength and mean wind speed. (Corr. coeff. = 0·91)

Fig. 15



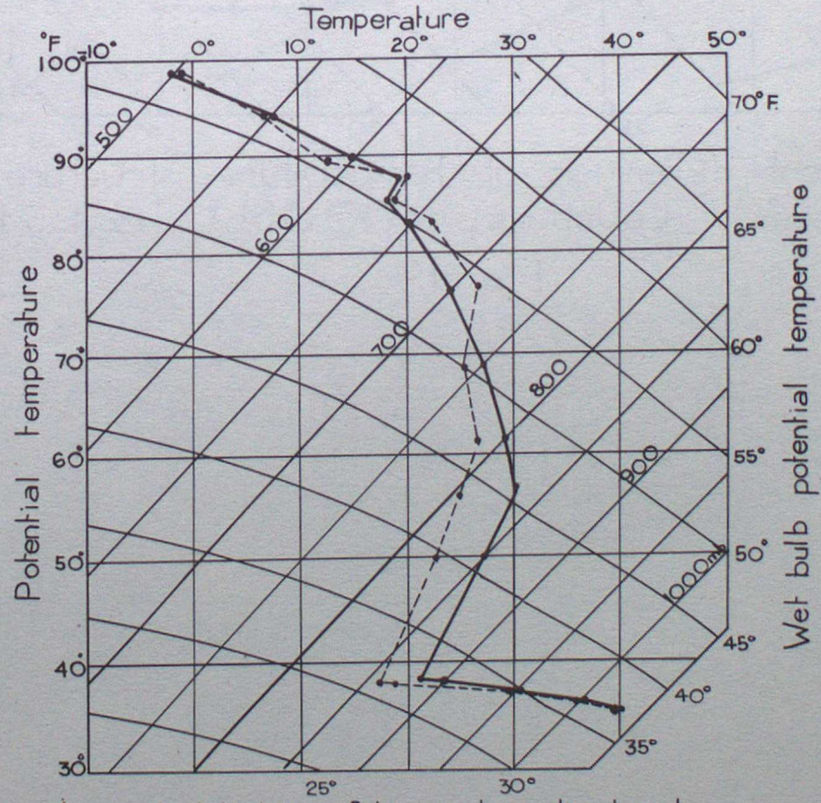
Relation between maximum vertical current and wavelength (Leuchars)

Fig. 16



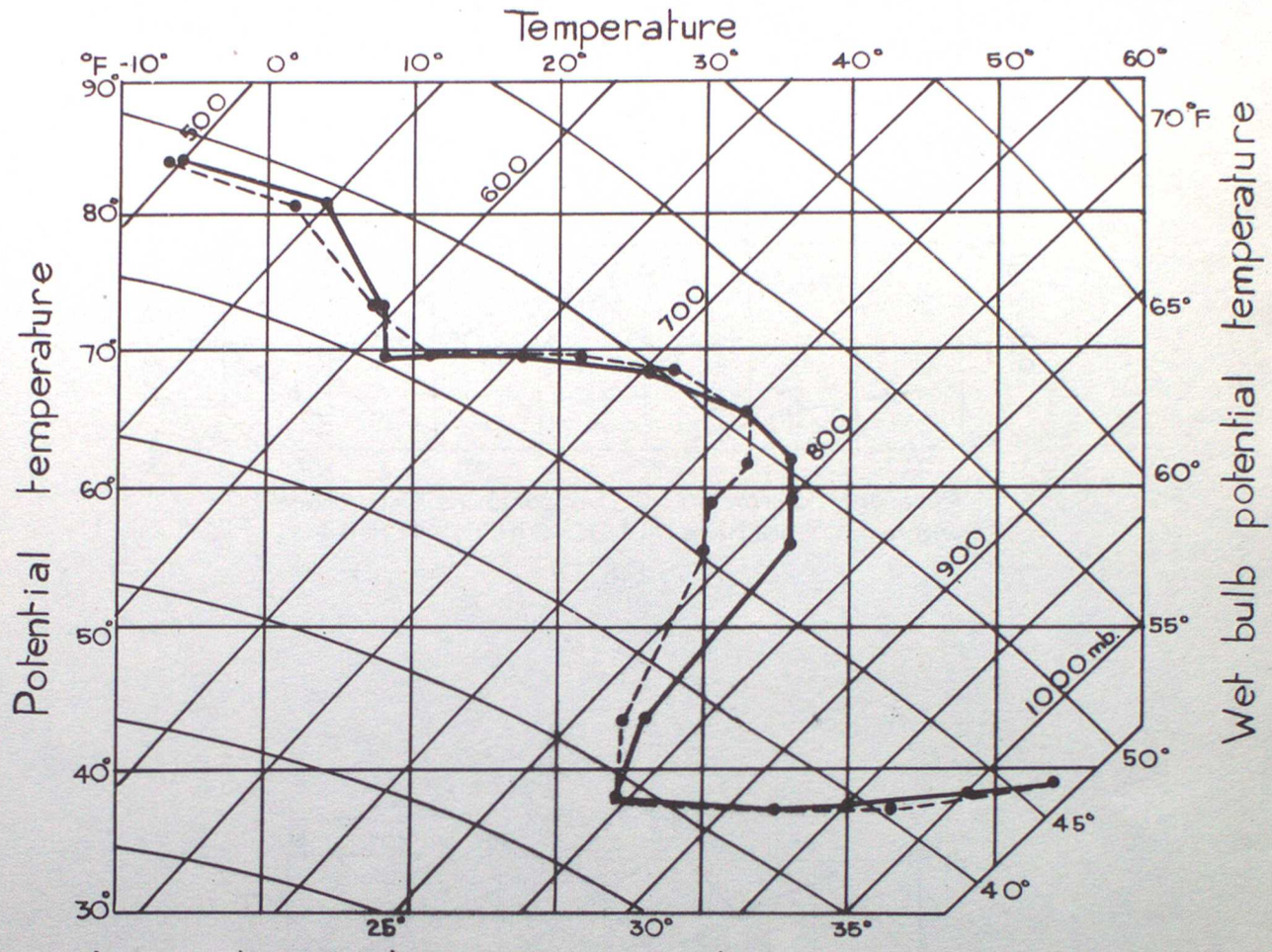
Apparent distortion of temperature structure by waves, Leuchars, 1400 G.M.T., 4:11:54

Fig. 17



Apparent distortion of temperature structure by waves, Leuchars 0200 G.M.T. 3:1:54

Fig. 18



Apparent distortion of temperature structure by waves, Leuchars, 1400 G.M.T 15:4:54

Fig. 19.